Mathematical Developments in Geophysical Fluid Dynamics

WORKSHOP 1:

Vortices and vorticity in geophysical flows

Avril 20 to 24, 2026 - IHP, Paris

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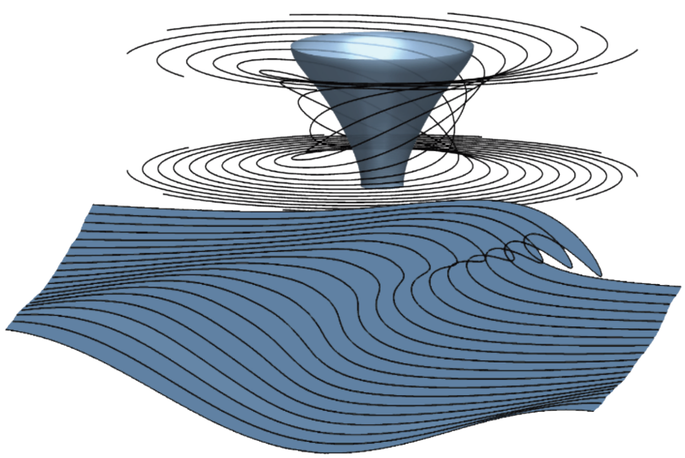

1 Schedule

Monday 20 April

Time	Speaker	*Title*
09:15 - 09:45	*Welcome coffee & registration*	
09:45 - 10:00	Introduction	
10:00 - 11:00	**Anna Mazzucato** [Clay Lecture CMI]	***Incompressible fluids with inflow/outflow***
11:00 - 12:00	**Geoffrey Vallis** [Maths-vives Chair]	***Superrotation in Planetary Atmospheres***
12:00 - 13:30	*Lunch*	
13:30 - 14:30	**Milton da Costa Lopes Filho**	***2D flows with random initial vorticity***
14:30 - 15:00	*(3 min each)*	*Poster presentations*
15:00 - 15:30	*Break*	
15:30 - 17:30		*Poster session*

Tuesday 21 April

Time	Speaker	*Title*
09:30 - 10:30	**Matthias Hieber**	***Coupled Atmosphere-Ocean Models***
10:30 - 11:00	*Break*	
11:00 - 11:30	**Luca Melzi**	*Quantitative stability of the Rossby–Haurwitz waves of degree two for the Euler equations on the sphere*
11:30 - 12:00	**Lyse Brichet**	*Eye of the cyclone: an experimental study*
12:00 - 13:30	*Group picture, then Lunch*	
13:30 - 14:30	**Tobias Barker**	***Quantitative classification of potential Navier-Stokes singularities beyond the blow-up time***
14:30 - 15:00	**Alan Riquier**	*Vorticity generation in water waves*
15:00 - 15:30	*Break*	
15:30 - 16:30	**Kerry Emanuel**	***Physics of Tropical Cyclones***
16:30 - 17:00	**Emeric Roulley**	*Filamentation near monotone zonal caps*
18:00 - 21:00	**Cocktail reception at the top of the Jussieu tower**	

Wednesday 22 April

Time	Speaker	*Title*
09:30 - 10:30	**Richard Rotunno**	***The Fluid Dynamics of Tornadoes***
10:30 - 11:00	*Break*	
11:00 - 11:30	**Basile Gallet**	*Transport properties of 2D turbulence*
11:30 - 12:00	**Delia Ionescu-Kruse**	*Surface Waves and Vortex Interaction*
12:00 - 13:30	*Lunch*	
13:30 - 14:30	**Beth Wingate**	***QG dynamics of the LANS-α model***
14:30 - 15:00	**Lars Eric Hientzsch**	*Vortex dynamics for the lake equations*
15:00 - 15:30	*Break*	
15:30 - 16:00	**Victor Navarro-Fernandez**	*Mixing with random cellular flows*
16:00 - 16:30	**Matthieu Ménard**	*Mean-field limits and the lake equations*
16:30 - 17:30	**Dongfen Bian**	***Boundary layers of Navier-Stokes system***

Thursday 23 April

Time	Speaker	*Title*
09:30 - 10:30	**Edward Johnson**	***Vorticity dynamics of coastal fronts***
10:30 - 11:00	*Break*	
11:00 - 11:30	**Jiao He**	*The vanishing limit of a rigid body*
11:30 - 12:00	**Bin Cheng**	*GFD on a fast rotating surface*
12:00 - 13:30	*Lunch*	
13:30 - 14:00	**David Meyer**	*Desingularisation of Vortex Sheets*
14:00 - 14:30	**Anthony Chen**	*A Lagrangian Vor-Div SWE Solver*
14:30 - 15:00	**Martin Donati**	*Leapfrogging of Vortex Rings*
15:00 - 15:30	*Break*	
15:30 - 16:00	**Haram Ko**	*On the stabilizing effect of rotation for 3D fluids*
16:00 - 16:30	**Henrik Latter**	*Instability of dusty rotating vortices*
16:30 - 17:00	**Dario Falcone**	*Cumulus Cloud Tilt and Ellipticity*
17:00 - 17:30	**Jason Barckicke**	*Kelvin waves along a free-surface vortex*

Friday 24 April

Time	Speaker	*Title*
09:30 - 10:30	**Andrew Gilbert**	***Geometry and vortex motion***
10:30 - 11:00	*Break*	
11:00 - 12:00	**Thierry Gallay**	***A point vortex in a half-plane***
12:00	*Lunch*	

2 Oral Presentations

Kelvin waves along a free-surface vortex
Jason BARCKICKE (MSC- Université Paris Cité)
Eric Falcon, Christophe Gissinger
Many flows feature a strong vorticity: tornadoes, hurricanes, aircraft wakes ... Vortices also play a key role in the energy cascade of classical and quantum turbulence. Kelvin waves are elementary perturbations of such vortices - helical waves propagating along the vortex line. Many theoretical works have predicted their behaviour but no experimental confirmation has been reported so far. By designing an original experimental set up, consisting in a bathtub vortex without rotation of the tank, we measured the Kelvin wave dispersion relation, with good resolution, for the first time. Their spectral signature is very rich and includes several wave modes. They all match the theoretical predictions. We then turn to investigating Kelvin wave turbulence - a phenomenon carrying energy towards small scales through nonlinear wave interactions. It paves the way to the experimental study of solitons and the dynamics of an array of vortices, whose collective motion is known as 'Tkachenko waves'.

Quantitative classification of potential Navier-Stokes singularities beyond the blow-up time)
Tobias BARKER (University of Bath)
It remains an open problem whether or not solutions to the 3D Navier-Stokes equations with smooth data remain smooth for all time. All previously known regularity criteria are formulated in times of a blow-up time (where the solution loses smoothness), which make it practically impossible to use such necessary conditions to test the viability of certain numerically computed candidates. Motivated by these issues, we give the first quantitative classification of potentially singular solutions at any given time in the region of potential blow-up times. The quantitative lower bounds prior to any potential blow-up time (and in the open vicinity of it) are in principle amenable to numerical testing.

Boundary layers of Navier-Stokes system
Dongfen BIAN (Beijing Institute of Technology)
The question of the stability of the boundary layers which appear as the viscosity of the fluid goes to zero is a classical question in Fluid Mechanics. In this talk I will discuss recent mathematical results on this question and in particular show that any shear layer is linearly and nonlinearly unstable provided the viscosity is small enough, and that the classical Prandtl boundary layers are always unstable.

Eye of the cyclone:an experimental study
Lyse BRICHET (Irphé)
Florian Rein, Ludivine Oruba, Emmanuel Dormy, Benjamin Favier, Michael Le Bars
One of the most characteristic flow in the atmosphere is the eye of tropical cyclones. Yet, its dynamic formation process is still poorly understood. This study focuses on a small number of parameters to improve our fundamental understanding of these dynamics. We have set up an experimental model in air to analyze the conditions for the formation of the eye of a cyclone in dry rotating convection. The setup consists of a rotating cylindrical tank, heated from below and maintained at a constant temperature on the side. The flow is measured using PIV in a vertical plane near the rotation axis. Within a well-defined parameter space, we observe a persistent descending flow along the rotation axis. Owing to the more extreme control parameters explored here compared with previous studies, the resulting eye is turbulent and non-axisymmetric. Despite this complexity, the eye formation is a robust feature of this idealised dry cyclone, providing that the Rossby number lies within a finite range

A Lagrangian Vor-Div SWE Solver
Anthony CHEN (University of Michigan)
In this work, I present a semi-Lagrangian solver for the spherical Shallow Water Equations, written in vorticity-divergence form. The solver makes use of a Biot-Savart law to compute the velocity, and is discretized on a tensor product Chebyshev grid on the cubed sphere. I also discuss some advantages and difficulties associated with using a semi-Lagrangian discretization, as well as the vorticity-divergence formulation of the SWE.

GFD on a fast rotating surface
Bin CHENG (University of Surrey)
Steve Schochet
We consider the rotating shallow water equations with small (planetary) Rossby and Froude numbers on a surface of revolution with variable Coriolis parameter having opposite signs at the poles. The large variation of the linear operator in the PDE is a possible mechanism of short-time instability as the small parameters tend to zero. However, we prove that such instability does not happen in this case: classical solutions satisfy uniform estimates on a time interval independent of the small parameters. The most novel part of our approach is to find the explicit formula of a modified Laplacian which commutes with the large linear operator of the system. Further, upon a unitary transformation, the solution converges strongly to a limit for which the governing system is identified.

2D flows with random initial vorticity
Milton DA COSTA LOPES FILHO (Universidade Federal do Rio de Janeiro)
Gautam Iyer, Helena Nussenzveig Lopes
We consider a random initial vorticity $\omega_0(x) = \sum_{n\in\mathbb{Z}^2} a_n\phi(x-n)$, where ϕ is bounded and compactly supported and $\{a_n\}$ are independent, uniformly bounded, mean 0, variance 1 random variables (in other words, ω_0 is an array of randomly weighted vortex blobs). We prove global well-posedness of weak solutions to the Euler equations in $\mathbb{R}^2$ for almost every such initial vorticity.

Leapfrogging of Vortex Rings
Martin DONATI (CNRS, LMA Poitiers)
Lars Eric Hientzsch, Christophe Lacave, Evelyne Miot
In this talk, we discuss recent developments concerning the motion of concentrated vortex rings. In particular, we outline the proof that, in the appropriate asymptotic regime, two concentrated coaxial vortex rings separated by a small distance exhibit the so-called leapfrogging motion. This highly singular regime lies beyond the scope of the standard tools of vorticity confinement. We introduce a new method to establish the persistence of mass concentration, and then implement a refined iterative scheme, combined with sharper estimates, to control the growth of the vorticity support. The choice of the asymptotic regime will be discussed, along with the technical limitations of our result.

Physics of Tropical Cyclones
Kerry EMANUEL (Massachusetts Institute of Technology)
Outside the immediate subdiscipline of tropical cyclone physics, it is widely held that these vortices are a mode of organization of deep, moist convection. In this lecture, I will dispel that idea and show that TC-like vortices arise from thermodynamic disequilibrium between a fluid and a solid or liquid surface, a disequilibrium that arises from the discontinuity in radiative emissivities between gases and solids or liquids. They can only arise in full turbulent flows in which the lower boundary is thermodynamically “rough”, so that heat flow is rate limited by turbulence and not by molecular diffusion. Phase changes are not necessary. I will focus on the energetics and dynamics of tropical cyclone-like vortices.

Cumulus Cloud Tilt and Ellipticity
Dario FALCONE (University of California, Davis)
Matthew Igel, Joseph Biello
Developing a tractable understanding of the interaction between cumulus clouds’ ellipticity and tilt from shear flows is crucial to expanding theories associated with squall line development and tradewind cumuli climatological feedbacks. Here,

we focus on the dynamic interplay between cloud-scale flows and shear flows. To perform this investigation, we implement a Kinematic Representation of Neutrally-buoyant Updraft Tori (KRoNUT) model for cloud-scale motions that allows for tilt and ellipticity. Using a moment closure technique, we then solve for the Dynamics of Neutrally-buoyant Updraft Tori (DoNUT) equations, a coupled non-linear system of ordinary differential equations which govern the temporal evolution of the parameters describing the intensity and geometry of a cloud-scale flow. Using this technique, we analytically and numerically investigate the behavior of the DoNUT equations to better appreciate how this additional geometry influences the life cycle of a cumulus cloud.

Transport properties of 2D turbulence

Basile GALLET (Université Paris Saclay, CEA Saclay)
Julie Meunier

The Kraichnan-Leith-Batchelor inverse energy cascade is a hallmark of 2D turbulence. Based on such phenomenology, the effective diffusivity of a 2D turbulent flow is dimensionally controlled by the energy flux and the large-scale friction coefficient only. Surprisingly, however, we show that such scaling predictions are invalidated by numerical solutions of the 2D Navier-Stokes equation forced at small wavenumber and damped by weak linear or quadratic drag. We derive alternate scaling-laws for the effective diffusivity based on the emergence of intense, isolated vortices causing spatially inhomogeneous frictional dissipation localized within the small vortex cores. The predictions quantitatively match DNS data. This study points to a universal large-scale organization of 2D turbulent flows in physical space, bridging standard 2D Navier-Stokes turbulence with large-scale geophysical turbulence.

Geometry and vortex motion

Andrew GILBERT (University of Exeter)
Steve Childress

A geometrical viewpoint of inviscid incompressible fluid dynamics highlights vorticity as the key field which generates the velocity field and is in turn transported, stretched and rotated, that is Lie-dragged, in the fluid flow. In this setting it is most natural to consider the velocity as a vector field, the momentum as a one-form (or co-vector) field, and the vorticity as a two-form field, making use of the metric and corresponding volume form. Such a view point is not only helpful in the abstract, but also gives practical ways of writing down the equations for vortex motion in a Lagrangian framework, where the coordinate system follows the evolution of a slender vortex. This talk will describe how one can write down the equations for vortex motion using such a coordinate system, which is general is both non-orthogonal and time-dependent. We will apply the framework to recover classic results on the motion of slender vortex rings.

The vanishing limit of a rigid body

Jiao HE (Laboratoire mathématique d'Orsay, University of Paris-Saclay)

We consider the evolution of a small rigid body in an incompressible viscous fluid filling the whole space R3 . When the small rigid body shrinks to a point in the sense that its density is constant, we prove that the solution of the fluid-rigid body system converges to a solution of the Navier–Stokes equations in the full space.Based on some Lp - Lq estimates of the fluid–structure semigroup and a fixed point argument, we obtain a uniform estimate of velocity of the rigid body. This allows us to construct admissible test functions which plays a key role in the procedure of passing to the limit.

Vortex dynamics for the lake equations

Lars Eric HIENTZSCH (KIT)
Christophe Lacave, Evelyne Miot

The lake equations arise as a 2D geophysical model describing the vertically averaged evolution of an incompressible inviscid 3D fluid in a domain with spatially varying topography. We rigorously derive the asymptotic dynamics of point vortices. Specifically, we show that initially sharply concentrated vorticity remains concentrated in a suitable sense. The vortices' trajectories are proven to follow the level lines of the depth function. The result holds for generic concentrated initial data.

Coupled Atmosphere-Ocean Models

Matthias HIEBER (TU Darmstadt, Germany)

In this talk we discuss coupled atmosphere-ocean models described by compressible/incompressible primitive equations subject to either deterministic nonlinear winddriven boundary conditions or stochastic boundary conditions on the interface.

Surface Waves and Vortex Interaction

Delia IONESCU-KRUSE ("Simion Stoilow" Institute of Mathematics of the Romanian Academy (IMAR))

The interaction between free-surface waves and localized vorticity structures is a fundamental problem in fluid dynamics, with relevance to geophysical flows. We study this problem by using the general framework for 2D water waves with arbitrary vorticity developed by Ionescu-Kruse and Ivanov (JDE, 2023). In the small-amplitude long-wave Boussinesq and KdV regimes, we derive coupled evolution equations for the free surface and the vortex dynamics. Our analysis shows that the interaction with the vortex does not destroy the surface solitary waves and, for a significant range of the vortex strength, the solitary waves remain practically unaffected. This observation leads to a further simplification of the model, in which the vortex motion beneath propagating solitons is described by a decoupled system of ODEs, capturing the qualitative features of the interaction. Analytical results are complemented by numerical simulations (see Ionescu-Kruse, Ivanov, Todorov, J. Nonlinear Sci, 2026).

On the stabilizing effect of rotation for 3D fluids
Haram KO (Brown University (U.S.A.))
Benoît Pausader, Takada Ryo, Klaus Widmayer
It is well known to geophysicists that the rotation of the background can bring about different phenomena not observed in steady fluids. In this talk, I will explain from a mathematical point of view what the rotation introduces in 3D Euler/Navier-Stokes equation, and talk about recent results of how the rotation can enhance the stability (a) in incompressible fluid with high Reynolds number and (b) in compressible Euler equation.

Instability of dusty rotating vortices
Henrik LATTER (DAMTP, University of Cambridge)
Nathan Magnan
We explore the various instabilities that potentially assail vortex cores in the presence of rotation and/or radially drifting solids. We first revisit the classical elliptical instability of rotating purely hydrodynamic vortices, showing that, in certain natural limits, the problem is governed by the 'inverted Matthieu equation', which provides a novel and remarkably simple way to conceptualise the parametric instability of the non-modal inertial waves. Second, we construct idealised vortex solutions involving a drifting dust fluid and a two-way drag force, using a multiple scales approach and the presence of a conserved 'dusty potential vorticity'. These solutions are subject to new small-scale instabilities that can be categorised as 'resonant drag instabilities', involving a coupling between the dust advective mode and inertial waves, though as both are non-modal, the classical theory needs significant reworking.

Incompressible fluids with inflow/outflow
Anna MAZZUCATO (Penn State University)
I will discuss recent results concerning the well-posedness of the Euler equations with inflow/outflow (injection/suction) at permeable boundary and the vanishing viscosity limit for both Navier-Stokes and the Boussinesq system.

Quantitative stability of the Rossby—Haurwitz waves of degree two for the Euler equations on the sphere
Luca MELZI (Imperial College London)
Matias G. Delgadino
The motion of an incompressible, ideal fluid is described by the Euler equations. Choosing the unit sphere $\mathbb{S}^2$ embedded in $\mathbb{R}^3$ as the domain of interest, the Euler equations represent a suitable model for stratospheric flows. Among such flows, of particular importance in atmospheric dynamics are the Rossby—Haurwitz waves, that are observed in the stratosphere of the Earth and other planets, such as Jupiter, Saturn, Uranus, and Neptune. In this joint work with Matias G. Delgadino (https://arxiv.org/abs/2509.16156), we show that the degree-2 Rossby—Haurwitz travelling waves on the Euler equations on $\mathbb{S}^2$ are orbitally stable. Our proof is short, quantitative, and conceptually easy to follow.

Mean-field limits and the lake equations
Matthieu MÉNARD (Université Paris Cité)
In this talk, we will investigate the mean-field limit of a system of differential equations introduced by Richardson to model the evolution of small concentrated vortices in a lake of non-constant depth.
Namely, we will show that when the number of vortices becomes very large, their distribution converges to the solution of the lake equations. The latter can be seen as a variant of the planar Euler equations that take into account the topography of the lake.
This result is based on a modulated energy approach introduced by Duerinckx and Serfaty that we adapt to deal with the heterogeneity of the lake interactions.

Desingularisation of Vortex Sheets
David MEYER (ICMAT (will change to MPI Leipzig at 1st of April))
Alberto Enciso, Antonio Fernandez
We show how to regularize vortex sheets by means of smooth, compactly supported vorticities that asymptotically evolve according to the Birkhoff–Rott vortex sheet dynamics. More precisely, consider a vortex sheet initial datum ω^0_{sing}, which is a signed Radon measure supported on a closed curve. We construct a family of initial vorticities $\omega^0_\epsilon \in C^\infty_c(\mathbb{R}^2)$ converging to ω^0_{sing} distributionally as $\epsilon \to 0^+$, and show that the corresponding solutions $\omega_\epsilon(x,t)$ to the 2D incompressible Euler equations converge to the measure defined by the Birkhoff–Rott system with initial datum ω^0_{sing}. The regularization relies on a layer construction designed to exploit the key observation that the Kelvin–Helmholtz instability has a strongly anisotropic effect: while vorticities must be analytic in the "tangential" direction, the way layers can be arranged in the "normal" direction is essentially arbitrary.

Mixing with random cellular flows
Victor NAVARRO-FERNANDEZ (Imperial College London)
Christian Seis (Universität Münster)
We study a passive scalar equation on the two-dimensional torus, where the advecting velocity field is given by a cellular flow with a randomly moving center. We prove that the passive scalar undergoes mixing at a deterministic exponential rate, independent of any underlying diffusivity. Furthermore, we show that the velocity field enhances dissipation and we establish sharp decay rates that, for large times, are deterministic and remain uniform in the diffusivity constant. Our approach is purely Eulerian and relies on a suitable modification of Villani's hypocoercivity method, which incorporates a larger set of Hörmander commutators than Villani's original method.

Vorticity generation in water waves
Alan RIQUIER (Fields Institute, Toronto)
Emmanuel Dormy
We propose to discuss the relevance of the irrotationality assumption commonly made to obtain reduced water waves models (Shallow Water/Saint-Venant, Korteweg-de Vries, Green-Naghdi, etc.). To do so, we investigate the asymptotic behaviour of two boundary layers associated with oceanic flows: one appearing in the vicinity of the free surface and the other lying at the bottom boundary. This is achieved numerically approximating the solution to the Navier-Stokes equations using the finite-element method on a moving mesh.

The Fluid Dynamics of Tornadoes
Richard ROTUNNO (NSF NCAR)
Observations, laboratory experiments and numerical simulations provide the basis for the theoretical fluid dynamics of the tornado. Tornado fluid dynamics is best understood in terms of the dynamics of several subproblems: the two-cell vortex, the boundary-layer beneath a potential vortex, the formation of an end-wall vortex and its vortex breakdown. Vortex breakdown involves further understanding of super- and sub-critical columnar vortices and their connection to each other through the concept of conjugate states. The bulk of this presentation follows Rotunno (2013 Ann Rev Fluid Mech). Recent progress in tornado theory comes in the form of present-day, high-resolution numerical simulations of tornadic supercell thunderstorms which now have the resolution to include surface friction. Many of the characteristics of the above-described tornado fluid dynamics are being reported in the present generation of tornadic supercell simulations (Rotunno and Bluestein 2024 Rep Prog Phys).

Filamentation near monotone zonal caps
Emeric ROULLEY (Università degli Studi di Milano)
Gian Marco Marin
We study the Euler equations on the rotating unit sphere, focusing on the dynamics of vortex caps, i.e. piecewise constant absolute vorticity. Leveraging the area stability of monotone, longitude-independent profiles, we demonstrate that certain ill-prepared initial data within the vortex cap class exhibit an instability characterized by the growth of the interface perimeter. These configurations are nearly equivalent in area to a zonal vortex cap but are perturbed by a localized latitudinal bump. By comparing the longitudinal flows at points along the zonal interface and within the bump region, we track the induced stretching and capture the underlying instability mechanism.

Superrotation in Planetary Atmospheres
Geoffrey VALLIS (University of Exeter)
Jets and Superrotation are ubiquitous features in some, but not all, planetary atmospheres. In particular superrotation occurs in slowly rotating terrestrial atmospheres (like Venus), in rapidly rotating gas giants (like Jupiter) and in some exoplanets. In this talk I'll discuss the various mechanisms giving rise to superrotation and their connection to potential vorticity.

QG dynamics of the LANS-alpha model
Beth WINGATE (University of Exeter)
Lulabel Seitz
The Lagrangian-Averaged Navier-Stokes-α (LANS-α) model, a fluid dynamics model based on energy-conserving modifications to nonlinear advection, can produce more energetic simulations than standard models, leading to improved fidelity (e.g., in ocean models). However, comprehensive understanding of the mechanism driving this energetic enhancement has proven elusive. To address this, we derive the fast quasi-geostrophic limit of the three-dimensional, stably-stratified LANS-α equations. This provides both the slow, balanced flow and the leading-order fast wave dynamics. Analysis of these wave dynamics suggests that an explanation for the energetic enhancement lies in the dual role of the alpha parameter: increasing α regularizes the dynamics and simultaneously generates a robust landscape of wave-wave resonant interactions where α modifies the role of Burger number. We discuss this using simulations and connect our results to numerical stability issues.

3 Posters

The lack of exponential stability

Monia BEL HADJ SALAH (Partial Differential Equations)

The lack of exponential stability for a weakly coupled wave equations through a variable density term.

In this paper, we consider a system of two wave equations coupled through zero order terms. One of these equations has an internal damping, and the other has a boundary damping. We investigate stability properties of the system according to the variable strings densities. Indeed, our main result is to show that the corresponding model is not exponentially stable using a spectral theory which forms the center of this work. Otherwise, we establish a polynomial energy decay rate.

Rotating turbulence: 2D or not 2D?

Sébastien GOMÉ (Technion)

Anna Frishman

Three-dimensional flows under solid-body rotation produce 3D inertial waves, while also sustaining emergent two-dimensional structures and favoring domain-scale flows. This interplay raises a fundamental question: why and when are 2D flows sustained even when only 3D waves are excited? We show that near-resonant interactions between 3D waves and a large-scale 2D flow impose an additional conservation law: the helicity of the waves, by sign. This hidden sign-definite invariant constrains the waves to transfer their energy to large-scale 2D motions. However, as rotation increases, resonance conditions become more restrictive and the energy transfer from 3D to 2D progressively vanishes, leading to a transition from 2D-dominated to 3D-dominated wave turbulence. We develop a perturbative framework capturing analytically the 3D–2D energy transfer as a function of rotation, Reynolds number and domain geometry, which agrees well with numerical simulations of the primitive equations.

The unusual dynamics of heated vortices

Bernard LEGRAS (Laboratoire de Météorologie Dynamique, ENS-PSL & CNRS)

Aurélien Podglajen, Mariem Rezig, Pasquale Sellitto

Spectacular rising long-lived anticyclonic smoke vortices have been discovered in the stratosphere after large wildfires. The dynamics of these vortices is very unusual. A key feature is the generation of a vertical tracer front in potential temperature. In turn this tracer front generates a layer of near zero potential vorticity which is associated with a temperature dipole and anticyclonic rotation. The heating is produced by solar absorption and radiative damping of the temperature anomaly. We discuss a minimal axisymmetric model in the inviscid but diabatic limit where the tracer follows a Burgers like weak solution and where temperature and wind are inverted from the singular potential vorticity which, in this model, is entirely constrained by the tracer. The competing effects of heating, damping and the role of the lateral scales are investigated. The solutions are used to interpret the results of observations and numerical simulations using the state of the art WRF model.

Instability result to the NS equations

Tien-Tai NGUYEN (Vietnam National University, Hanoi)

In this paper, we investigate the instability of the trivial steady states to the incompressible viscous fluid with Navier-slip boundary conditions. For the linear instability, the existence of infinitely many normal mode solutions to the linearized equations is shown via the operator method of Lafitte and Nguyen (2022). Hence, we prove the nonlinear instability by adapting the framework of Desjardins and Grenier (2003) studying some classes of viscous boundary layers to obtain two separated solutions at escaping time. Our work performs a different approach from that of Ding, Li and Xin (2018).

Results on flow-induced oscillations

Clara PATRIARCA (Université Libre de Bruxelles)

Giovanni Galdi, Denis Bonheure

We show some results concerning the mathematical analysis of flow regimes past a bluff body as the incoming flow intensity increases. Experiments show that at a critical threshold the flow transitions from steady laminar to unsteady periodic behaviour, characterised by vortex shedding and oscillatory wake. This transition strongly affects the body dynamics, leading to vortex-induced vibrations. When the shedding frequency matches the structure's natural frequency, resonance might cause large-amplitude oscillations, potentially causing fatigue and structural failure. To study these transitions, we consider a coupled system, consisting of an incompressible Navier-Stokes fluid interacting with an undamped structure in an exterior domain. The motion of the system is driven by the uniform flow of the liquid, far away from the body. We show for this model the existence and uniqueness of equilibria, and the absence of any resonant phenomenona.

Slow–Fast Separation for Large-Timesteps
Juliane ROSEMEIER (Freie Universität Berlin)
Slow–Fast Separation for Large-Timestep Integration of Rotating Multiscale Flows

Rotating geophysical flows such as the Rotating Shallow Water equations and the Rotating Boussinesq equations exhibit a multiscale structure in which fast oscillatory wave modes interact with slow dynamics. These rapid oscillations impose severe timestep restrictions on explicit schemes, even when the relevant evolution is slow.

We introduce a transformation that separates slow and fast components in highly oscillatory systems. The approach is illustrated on ordinary differential equations, including the Van der Pol oscillator, where the leading-order dynamics evolves on a slow scale. In a second example, the transformed system becomes entirely slow. This separation enables the construction of time-stepping methods that remain stable and accurate for timesteps larger than the fast wave period.

Waves in rotating stratified vortices
Jérémie VIDAL (CNRS, ENS de Lyon, Univ. Lyon 1)
Pancake-like vortices are often generated by turbulence in geophysical flows. Here, we study the inertia-gravity oscillations that can exist within such geophysical vortices, due to the combined action of rotation and gravity. We consider a fluid enclosed within a triaxial ellipsoid, which is stratified in density with a constant Brunt–Väisälä frequency (using the Boussinesq approximation) and uniformly rotating along a (possibly) tilted axis with respect to gravity. The wave problem is then governed by a mixed hyperbolic-elliptic equation for the velocity. As in the rotating non-stratified case, we find that the spectrum is pure point in ellipsoids with polynomial eigenvectors. Then, we characterise the spectrum using numerical computations (obtained with a bespoke Galerkin method) and asymptotic spectral theory. Finally, the results are discussed in light of natural applications (e.g. for Mediterranean eddies or Jupiter's vortices).

Matrix models in spherical hydrodynamics
Milo VIVIANI (Scuola Normale Superiore)
Klas Modin
Two-dimensional (2-D) incompressible, inviscid fluids produce fascinating patterns of swirling motion. How and why the patterns emerge are long-standing questions, first addressed in the 19th century by Helmholtz, Kirchhoff, and Kelvin. Countless researchers have since contributed to innovative techniques and results, but the overarching problem of swirling 2-D motion and its long-time behavior remains largely open. Here we advocate an alternative view-point that sheds light on this problem via a link to isospectral matrix flows. The link is established through V. Zeitlin's beautiful model for the numerical discretization of Euler's equations in 2-D. When considered on the sphere, Zeitlin's model enables a deep connection between 2-D hydrodynamics and unitary representation theory of Lie algebras as pursued in quantum theory. Furthermore, we show how Zeitlin's model enables to investigate global phenomena in 2-D spherical hydrodynamics, which have direct impact in GFD.

4 Sponsors