Mathematical Developments in Geophysical Fluid Dynamics

WORKSHOP 3:

Idealised mathematical models for geophysical flows

June 29 to July 3, 2026 - IHP, Paris

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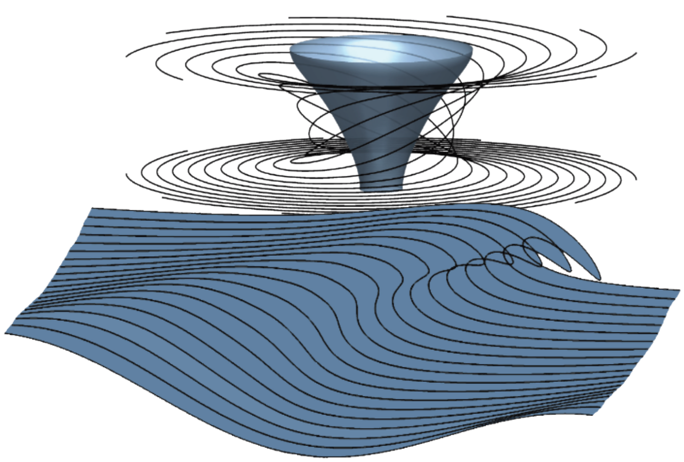

1 Schedule

Monday 29 June

Time	Speaker	*Title*
09:15 - 09:45	*Welcome coffee & registration*	
09:45 - 10:00	Introduction	
10:00 - 10:50	**Andrej Zlatos**	***Stable regime singularity for the Muskat problem***
10:50 - 11:20	**Maria Kazakova**	*A model for breaking of surface waves*
11:20 - 12:10	**Philip Marcus**	***A Great Red Spot: Models vs Observations***
12:10 - 13:30	*Lunch*	
13:30 - 14:20	**Helena Nussenzveig Lopes**	***On local energy conservation***
14:20 - 15:10	**Andy Jackson**	***Progress towards the inviscid dynamo***
15:10 - 15:40	*Break*	
15:40 - 16:40	*(2 min each)*	*Poster presentations*
16:40 - 18:00		*Poster session*

Tuesday 30 June

Time	Speaker	*Title*
09:30 - 10:20	**Paola Cessi**	***Ocean Transit Times***
10:20 - 10:50	*Break*	
10:50 - 11:20	**David Gérard-Varet**	*Reduced models for bubbly flows*
11:20 - 12:10	**Ángel Castro**	***Solutions near monotone vortices***
12:10 - 13:30	*Group picture, then Lunch*	
13:30 - 14:00	**Pascale Garaud**	*Stratified Turbulence*
14:00 - 14:30	**Frédéric Charve**	*3D-2D asymptotics for the rotating MHD*
14:30 - 15:00	**Toby Wood**	*The deep-seated solar dynamo*
15:00 - 15:30	*Break*	
15:30 - 16:20	**Benoit Pausader**	***Linear Stability of small BGK waves***
16:20 - 16:50	**Benjamin Miquel**	*QG convection on the f-plane*
16:50 - 17:20	**Klaus Widmayer**	*Couette flow and rotation/stratification*

Wednesday 1 July [AI day]

Time	Speaker	*Title*
09:30 - 10:20	**Javier Gómez-Serrano**	***New frontiers of mathematics: doing numerics in the age of AI***
10:20 - 10:50	*Break*	
10:50 - 11:40	**Taraneh Sayadi**	***Data-driven reduced order models***
11:40 - 12:10	**Anna Guseva**	*Data-driven dynamo equations*
12:10 - 13:30	*Lunch*	
13:30 - 14:20	**Matthew Juniper**	***The elephant in the room: Probabilistic Machine Learning into physical models***
14:20 - 14:50	**Louis Thiry**	*Generative modeling of QG solutions*
14:50 - 15:20	*Break*	
15:20 - 16:10	**Peter Korn**	***Post-Geostrophy: Numerics, Computation, AI***
16:10 - 17:00	**Rupert Klein**	***Thoughts on Machine Learning***

Thursday 2 July

Time	Speaker	Title
09:30 - 10:20	**Theodore Drivas**	***Intermittency & dissipation in turbulence***
10:20 - 10:50	*Break*	
10:50 - 11:20	**Nick Pizzo**	*Exact solutions to Euler's equations*
11:20 - 12:10	**David Marshall**	***Simple model for the strength of the ACC***
12:10 - 13:30	*Lunch*	
13:30 - 14:20	**Sergey Gavrilyuk**	***Dispersive shock waves***
14:20 - 14:50	**Mei Ming**	*Water-waves problem with contact angles*
14:50 - 15:20	*Break*	
15:20 - 16:10	**Paul Milewski**	***Wave resonances in bounded domains***
16:10 - 16:40	**Francisco Beron Vera**	*Metriplectic ocean thermodynamics*
16:40 - 17:10	**Slim Ibrahim**	*Boundary layer around a rotating cylindre*

Friday 3 July

Time	Speaker	Title
09:30 - 10:20	**David Lannes**	***Boundary conditions and non kinematic free boundaries for wave-structure interactions***
10:20 - 10:50	*Break*	
10:50 - 11:20	**Paula Wulff**	*Scaling Trends in Rotating Convection*
11:20 - 12:10	**William Young**	***Polar Vortex Crystals***
12:10	*Lunch*	

2 List of posters

C brackets for shallow-water sloshing	Hamid Alemi Ardakani
On the size & shape of condensing steam	Bruno Andreotti
Scalings of Moist Convection under Climate Change	Gregory Dritschel
alpha-modified CL equations using GLM	Philomène Dufour
Counter-rotating spherical Couette flow	Juan Cruz Gonzalez Sembla
Long-time behavior of ocean toy models	Julien Guillod
Energetics in sorted density coordinates	Benedict Hatton
Conservative discrete flows	Antoine Leblond
Free Boundary 2D Navier-Stokes System	Paula Luna Velasco
Criticality and Navier-Stokes-Boussinesq	Maxence Mansais
Exponential Mixing on the unit sphere	Marc Nualart
On the Fritz John problem	Martin Oen Paulsen
On Stratosphere-Troposphere Coupling	Maria Reboredo Prado
Modelling transitional boundary layer	Maharun Nesa Shampa
Effects of basin wind-stress on the MOC	Jonathan Tessier
Equilibrium statistical physics of waves	Alexandre Tlili
Convection in rotating spherical shell	Laurette Tuckerman
Geostrophic internally heated convection	Yutong Zhang

3 Oral Presentations

Metriplectic ocean thermodynamics
Francisco BERON VERA (University of Miami)
We present a metriplectic formulation of a reduced model for the upper ocean. The model is valid at low frequencies, includes a single layer with lateral inhomogeneity and uniform stratification, and is thermodynamically consistent - that is, it conserves energy while producing entropy. The evolution of any functional of the model variables (horizontal velocity, layer thickness, and buoyancy's vertical average and gradient) is governed by its (Lie-)Poisson bracket with the Hamiltonian, plus a symmetric bracket with a Casimir that incorporates dissipation. The symmetric bracket is constructed in two ways: algebraically and using the metric on the flow domain, the latter justifying the term 'metriplectic bracket.' This is joint work with Erwin Luesink (University of Amsterdam).

Solutions near monotone vortices
Ángel CASTRO (ICMAT)
In this talk we will consider the existence of rotating solutions arbitrarily close (in some topology) to radial monotone decreasing vorticity for 2D Euler. In a paper by Bedrossian, Coti-Zelati and Vicol was shown that radial monotone decreasing vorticities are stable at the linear level, thus, our result shows that this phenomenon can break even for small perturbation. The problem is related with the stability of shear flows and the existence of stationary and traveling waves solution near them. We also review some results on this topic.

Ocean Transit Times
Paola CESSI (Scripps Institution of Oceanography, University of California, San Diego)
Lagrangian transit times on basin to planetary scales are controlled by the interplay of multiscale processes. The primary advective timescale is set by throughflow currents, such as interhemispheric western boundary currents. Dispersion by mesoscale eddies introduces fluctuations that erase memory and enhance dispersion, widening the transit-time distribution. The tortuous paths of Lagrangian parcels, particularly within ocean gyres, significantly enhance dispersion beyond the levels attributed to mesoscale eddies alone. Additionally, trapping by ocean gyres leads to multimodal distributions of Lagrangian transit times. These processes are illustrated in three complementary contexts: eddy-permitting ocean state estimates, simplified spatially extended three-dimensional flows and diffusively coupled two-dimensional pipe models.

3D-2D asymptotics for the rotating MHD
Frédéric CHARVE (LAMA – Université Paris-Est Créteil)
joint work with Van-Sang Ngo
In this joint work with Van-Sang Ngo, we consider the 3D-rotating magnetohydrodynamic (MHD) system.

We begin this talk by providing a few examples of penalized geophysical models similar to the incompressible Navier-Stokes system, and which converge (when the small penalization parameter goes to zero) towards a limit system that can be easily seen to be incomplete. Reaching a more complete limit requires adding to some classical initial data a non-conventional component linked to the special structure of the limit system.

Then we study (for both weak and strong solutions) the asymptotics when the Rossby number goes to zero (i.e. for strong rotation) of the 3D-rotating (MHD) system when the initial velocity and magnetic field both feature some 2D-part (i.e. depending only on the horizontal space variables).

We show this limit is the 2D-MHD system *with three components* supplemented with an additional 3D magnetic field transported by the 2D limit velocity.

Intermittency & dissipation in turbulence
Theodore DRIVAS (Stony Brook University)
joint work with Luigi De Rosa
Intermittency is a remarkable and robust feature of three-dimensional turbulence for which we still lack explanation from first principles. It will be shown how a dissipation with a non-trivial lower-dimensional part induces a quantitative intermittent regularity on the weak solution.

Stratified Turbulence
Pascale GARAUD (UC Santa Cruz)
I will present recent results comparing asymptotic models of stratified turbulence (Chini et al. 2022) with the results of numerical experiments (Garaud et al. 2024). Stratified turbulence becomes highly intermittent in the limit of strong stratification. Key results include a characterization of the volume fraction of the fluid occupied by turbulent patches,

and scaling laws for the rms vertical velocity of the flow inside and outside of the patches, as functions of the Froude, Reynolds, and Prandtl numbers.

Dispersive shock waves
Sergey GAVRILYUK (Aix-Marseille Université et UMR CNRS 7343 IUSTI)
The objective of this talk is to describe the solitary wave of largest amplitude in the dispersive shock appearing in the solution of the Riemann problem for dispersive equations describing non-linear long dispersive waves, in particular, the Benjamin-Bona-Mahony equation and Serre-Green-Naghdi equations. Such a large-amplitude solitary wave is the leading wave of the corresponding dispersive shock. Its speed and amplitude are defined analytically through the solitary limit of the corresponding Whitham modulation equations. In such a limit, Whitham's equations form a system of quasi-linear equations for which Riemann's invariants can be determined. The numerical results are in accordance with the analytical prediction.

Reduced models for bubbly flows
David GERARD-VARET (Université Paris Cité)
Joint work with Cosmin Burtea
One main issue in the analysis or simulation of immiscible two phase flows is the description of the moving interface between the two phases, especially when this interface has several connected components. In the case of bubbly liquids, in which the gas bubbles do not deviate much from simple geometries (spheres, ellipsoids), one may try to reduce the complexity of the model by imposing shape constraints on the bubbles through their evolution. This imposes in turn to relax the usual constraints at the gas/liquid interface. We shall present a method to determine the appropriate relaxed interface conditions, based on a least action principle.

New frontiers of mathematics: doing numerics in the age of AI
Javier GOMEZ SERRANO (Brown University)
In this talk I will discuss recent results in Mathematics+AI. I will describe several concrete instances in which AI systems have contributed to genuine mathematical and scientific results, ranging from the discovery of new objects to the optimization of the numerical kernels that underpin modern computation. A recurring theme is that these tools do not replace mathematical reasoning but extend the range of problems on which it can be brought to bear, often by searching spaces that are too large or too irregular for human intuition alone.

Data-driven dynamo equations
Anna GUSEVA (Polytechnic University of Catalonia)
joint work with C. Skene, S. Tobias
Many low-mass stars like the Sun host periodic, oscillatory magnetic fields that lead to variable levels of stellar activity and variations of space weather, affecting habitability and detection of exoplanets. Due to the intrinsic difficulties of modelling stellar magnetohydrodynamics at all scales, realistic numerical simulations of this process are very challenging and their reduced-order models of oscillatory dynamos are of interest. In this work, we develop a framework to recover such models directly from numerical data using a combination of Hankel Dynamic Mode Decomposition (DMD) to identify magnetic structures, and Sparse Identification of Nonlinear Dynamics (SINDy) to model their dynamics, and compare it to classic mathematical method of weakly nonlinear analysis (WNL). We implement this approach on a one-dimensional idealized mean-field dynamo model parametrizing the main components of convective dynamo in a low-mass star, helical convection and differential rotation.

Boundary layer around a rotating cylindre
Slim IBRAHIM (University of Victoria, Department of Mathematics and Statistics)
joint work with Yasunori Maekawa
In this talk, I will first review the boundary layer problem and flow separation in a viscous incompressible fluid past a rigid cylindrical obstacle undergoing constant, but fast rotation (compared to a uniform background flow). Then, I will show how to solve the boundary layer equations, give a solvability criterion for the matched asymptotic expansion, and compare our findings with the geometric Feynman–Lagerstrom criterion recently revisited by Drivas–Iyer–Nguyen. New features seem to arise from the competition between the vorticity production and rotation-induced tangential transport, leading to boundary layer behaviour distinct from the classical Prandtl or Stokes layers.

Progress towards the inviscid dynamo
Andy JACKSON (Institute for Geophysics, ETH Zurich)
joint work with Longhui Yuan, Philippe Marti and Jiawen Luo
Earth's magnetic field is believed to be generated in the metallic outer core through a process known as geodynamo.

Direct numerical simulation (DNS) of geodynamo has successfully reproduced many features of the Earth's field. Still, even the state-of-the-art simulations have a much higher viscosity than the Earth's outer core. Taylor (1963) proposed a reduced model by neglecting inertia and viscous force. A modified model that partially re-introduces the inertia term back is termed the torsional wave (TW) dynamo model, since it admits torsional oscillations, a special type of Alfven wave. In this study, we present progress to date in our studies of inviscid dynamo simulation.

The elephant in the room: Probabilistic Machine Learning into physical models

Matthew JUNIPER (University of Cambridge)

John von Neumann is often quoted as saying "with four parameters I can fit an elephant, and with five I can make him wiggle his trunk." The implication seems to be that physical models should contain only a handful of parameters. A century later, however, we seem happy to use physics-agnostic neural networks containing millions of parameters. What would von Neumann say? How should physical modellers respond?

In this talk, I will show that von Neumann's quote is more nuanced than it sounds. I will then frame a response within a Bayesian framework, in which physical principles such as conservation of mass, momentum, and energy are treated as high quality prior information, with quantified uncertainty, expressed as PDEs or low order models. The information content of data can then be quantified and the likelihood of different candidate models can be compared after the data arrives. I will show how Bayesian inference becomes computationally tractable when combined with adjoint methods. I will demonstrate this through assimilation of 3D Flow-MRI data in complex geometry into Finite Element CFD. The main message of the talk is "keep the physics in the model if you can."

A model for breaking of surface waves

Maria KAZAKOVA (Univ. Savoie Mont Blanc, CNRS, LAMA)
joint work with Yen Chung Hung, Gaël Richard, Julien Chauchat

A shallow water model for the propagation and breaking of surface waves is proposed in the form of a hyperbolic system of conservation laws, with dispersive effects introduced through a relaxation term and a localized dissipative term. The latter is activated in regions where a new breaking criterion is satisfied. The objective is to obtain a simple mathematical and numerical structure while capturing the main features of wave breaking. The governing equations, the associated breaking criterion, and the numerical strategy used for their approximation are presented. Particular attention is paid to the persistence of the dissipation once activated, and to its influence on the behaviour of solutions. Several test cases illustrate the properties of the model.

Thoughts on Machine Learning

Rupert KLEIN (Freie Universität Berlin)

Techniques of machine learning (ML) find a rapidly increasing range of applications touching upon many aspects of everyday life. They are also used with enthusiasm to close gaps in our scientific knowledge by data-based modeling. I have followed these developments with interest, concern, and mounting disappointment. When these technologies take over decisive functionality in safety-critical applications, we should know how to guarantee their compliance with pre-defined guardrails. Moreover, when they are utilized as building blocks in scientific research, it would violate scientific standards if these building blocks were used without a thorough understanding of their functionality, including inaccuracies, uncertainties, and other pitfalls. In this context, I will juxtapose (a subset of) deep neural network methods with the family of entropy-optimal ML techniques developed recently by Illia Horenko (RPTU Kaiserslautern-Landau) and colleagues.

Post-Geostrophy: Numerics, Computation, AI

Peter KORN (Max Planck Institute for Meteorology)

Ocean climate modelling can now resolve geostrophic turbulence rather than merely parametrize it. With this step the ocean has taken the stage as a genuinely turbulent fluid - a remarkable achievement that also marks the end of the "geostrophic era." That ending is equally a beginning, and one that falls in line with the technological shift toward GPU and AI computing. This talk describes new computational approaches to ocean modelling, new experimental strategies and reflects on how machine learning can be integrated into this endeavour.

Boundary conditions and non kinematic free boundaries for wave-structure interactions

David LANNES (Université de Bordeaux)

The description of waves, through the water waves equations or simpler asymptotic models (such as the nonlinear shallow water equations or the Boussinesq system) is well understood in a domain without boundaries. In the case of wave-structure interactions, such as the dynamics of the shoreline or of floating objects, the free surface has a boundary formed by the contact line between the surface of the fluid and the surface of the solid. The presence of this boundary induces new difficulties such as the derivation and analysis of boundary conditions but also the analysis of the motion of the boundary itself. In this talk we review some known results on the treatment of boundary conditions for hyperbolic systems (such as the nonlinear shallow water equations), and propose some extensions motivated by wave-structure interactions. We will comment also on the treatment of boundary conditions for dispersive perturbations of hyperbolic systems (such as the Boussinesq equations) and introduce the notion of dispersive boundary layers. Finally, we will comment on the dynamics of the contact line, which is not always a kinematic boundary condition in the sense that a particle located on the contact line can detach from it.

Basque translation:
Uhin-egitura elkarrekintzetarako muga-baldintzak eta muga aske ez-zinematikoak
Uhinen deskribapena, ur-uhinen ekuazioen edo eredu asintotiko sinpleagoen bidez (sakonera txikiko uraren ekuazio ez-linealak edo Boussinesq sistema, adibidez), ondo ulertzen da mugarik gabeko domeinu batean. Uhin-egitura elkarrekintzen kasuan, hala nola itsasertzaren edo objektu flotatzaileen dinamika, surfaze libreak fluidoaren surfazea eta solidoaren surfazearen arteko kontaktu-lerroak osatutako muga bat du. Muga horren presentziak zailtasun berriak eragiten ditu, hala nola muga-baldintzen deribazioa eta analisia baina baita mugaren beraren higiduraren analisia ere. Hitzaldi honetan, sistema hiperbolikoetarako muga-baldintzen tratamenduari buruzko emaitza ezagun batzuk berrikusiko ditugu, eta uhin-egitura elkarrekintzek eragindako hedapen batzuk proposatuko ditugu. Sistema hiperbolikoen perturbazio dispertsiboen muga-baldintzen tratamendua ere aipatuko dugu (Boussinesq-en ekuazioak, adibidez), eta muga-geruza dispertsiboen nozioa sartuko dugu. Azkenik, kontaktu-lerroaren dinamikari buruz hitz egingo dugu, ez baita beti muga-baldintza zinematikoa, kontaktu-lerroan kokatutako partikula batek linea horretatik aska dezakeen zentzuan.

A Great Red Spot: Models vs Observations
Philip MARCUS (University of California, Berkeley)
Joint work with Aidi Zhang, Sungkyu Kim, Imke de Pater, Mike Wong, Anton Ermakov, Chris Moeckel, Daniele Durantej.
With sufficient resolution, a range of numerically-computed 3D vortices can quantitatively reproduce the observed velocity field of Jupiter's Great Red Spot (GRS) at its cloud-tops. Due to the range of these solutions' properties below the visible cloud tops, no particular vortex can be claimed to uniquely model the GRS. However, by requiring that the computed vortices also reproduce new cloud-top temperature observations, a unique solution to the equations of motion is obtained. This GRS solution has a vertical thickness of only 1% of its east-west diameter, does not penetrate the underlying convection zone, and has a large Rossby number beneath its observable velocities, so it violates the quasi-geostrophic and shallow-water approximations. The 3D GRS is poorly approximated by 2D models, models based on potential vorticity dynamics, statistical models based on maximum entropy, and solitary wave models. Like an iceberg, upper surface observations of the GRS are misleading.

Simple model for the strength of the ACC
David MARSHALL (University of Oxford)
joint work with Xiaoming Zhai (University of East Anglia), James Maddison (University of Edinburgh), Julian Mak (Hong Kong University of Science and Technology)
The volume transport of the Antarctic Circumpolar Current (ACC) is described in textbooks as set by wind and buoyancy forcing. However, eddy-permitting numerical ocean models indicate minimal sensitivity of ACC transport to the wind stress. A new model - building on the recent GEOMETRIC parameterisation of mesoscale eddies - is developed relating ACC transport to three length scales divided by the residence time of Southern Ocean eddy energy (set by bottom drag), and independent of the wind stress. Observation-based estimates of the parameters give an ACC transport of realistic magnitude.

Wave resonances in bounded domains
Paul MILEWSKI (Penn State University)
joint work with Matthew Durey
Nonlinear surface gravity waves sloshing in a container of rectangular cross-section can behave very differently than those with other cross sections. Wave resonance is a mechanism by which energy is continuously exchanged between a small number of wave modes and is common to many nonlinear dispersive wave systems. They have been studied extensively over the past 60 years, almost always on domains that are large (or infinite) compared to the characteristic wavelength. In this case, the dispersion relation dictates that only quartic (4-wave) resonances can occur. In contrast, wave resonances in confined three-dimensional geometries have received relatively little attention, where, perhaps surprisingly, stronger 3-wave resonances of gravity waves can occur. We will present the results characterizing the configuration and dynamics of resonant triads in cylindrical basins of arbitrary cross sections. Extensions to internal waves and other geometries will also be discussed.

Water-waves problem with contact angles
Mei MING (Yunnan University, China)
We will talk about a weighted a priori energy estimate for the two dimensional water-waves problem with contact points

in the absence of gravity and surface tension and some related topics. When the surface graph function and its time derivative have some decay near the contact points, we show that there is corresponding decay for the velocity, the pressure and other quantities in a short time interval. As a result, we have fixed contact points and contact angles. To prove the energy estimate, a conformal mapping is used to transform the equation for the mean curvature into an equivalent equation in a flat strip with some weights. Moreover, the weighted limits at contact points for the velocity, the pressure etc. are tracked and discussed. Our formulation can be adapted to deal with more general cases.

QG convection on the f-plane
Benjamin MIQUEL (CNRS, Ecole Centrale Lyon)
joint work with A. Ellison, M. Calkins, K. Julien, E. Knobloch
Planetary cores and the subsurface oceans of icy moons are stirred by quasi-geostrophic turbulent convection. The transport (of heat, momentum, etc.) by the flow vary regionally with the colatitude, which coincides with the tilt angle between gravity and rotation. Here, we analyse rapidly rotating Rayleigh-Benard convection in a local model: the tilted f-plane. Employing non-orthogonal coordinates, we obtain a natural formulation for geostrophy and a set of governing equations for the non-hydrostatic quasi-geostrophic dynamics on the tilted f-plane (fNHQGE), valid in the asymptotic limit of rapid rotation. We conduct a systematic parametric study by varying the Rayleigh number and the tilt angle. As the tilt increases, the barotropic condensate transitions from large scale vortices (near the pole) to East-West jets (near the equator), with bistability at intermediate latitudes. Concomitantly, both heat transport and the vertical kinetic energy decrease monotonically with colatitude.

On local energy conservation
Helena NUSSENZVEIG LOPES (Universidade Federal do Rio de Janeiro)
joint work with Tobias Barker, Milton Lopes Filho
Anomalous dissipation, i.e. the non-vanishing of the dissipation term in the limit of zero viscosity, is a cornerstone of turbulence theory. In the case of periodic, two-dimensional, incompressible fluid flow there has been a lot of recent work in which anomalous dissipation is ruled out if the vorticity, which is the curl of velocity, is p-th power integrable or even a nonnegative Radon measure. The vanishing viscosity limit in domains with boundary is a classical open problem due mostly to the formation of large gradients near the walls. These may propagate into the bulk of the fluid through boundary layer separation and have the potential to yield anomalous dissipation. In this talk we will discuss the absence of local anomalous dissipation in the vanishing viscosity limit in a bounded domain.

Linear Stability of small BGK waves
Benoit PAUSADER (Brown University)
joint work with Dongfen Bian, Emmanuel Grenier, Wenrui Huang
We consider the Vlasov-Poisson system in a 1d periodic setting, and consider the stability of steady states. The simplest family corresponds to homogeneous steady states, and a lot of literature has been devoted to their study. When a Penrose-type criterion is satisfied, following works of Mouhot-Villani and later works, perturbations are damped, when the Penrose criterion is violated, the situation is more complicated and related to another class of steady states: the (inhomogeneous) BGK waves.

Exact solutions to Euler's equations
Nick PIZZO (University of Rhode Island)
joint work with Rick Salmon
Exact solutions to the two dimensional Euler's equations, on Euclidean and non-Euclidean surfaces, are presented in Lagrangian coordinates. These solutions arise due to a particle relabeling invariance, a subset of which, associated with particle label rotations, are shown to transform time independent solutions to time dependent solutions by these infinitesimal canonical transformations. The associated compatibility conditions of these maps restrict the label dependence to be harmonic maps from cartesian label space to these two dimensional surfaces, connecting the rotational relabelling symmetry with harmonic maps. Using a frame equation approach on the sphere, harmonic maps from the plane to the sphere are associated with a negative sinh Laplace equation and the associated family of these maps, which rotate the Hopf differential, are shown to generate the time evolution. Simpler solutions with label dependent rotations are also presented.

Data-driven reduced order models
Taraneh SAYADI (Conservatoire National Arts et Métiers)
Reduced-order models offer computationally efficient approximations of complex systems, enabling multi-query tasks in design and optimisation with low cost and sufficient accuracy. Data-driven strategies are particularly appealing when underlying models are inaccessible or too expensive to evaluate, and recent advances in AI-based architectures have

naturally entered this space. However, these architectures still face challenges when confronted with systems exhibiting variable dynamics, bifurcations, or chaotic behaviour. In this talk, we present a shift in perspective that unifies complex dynamical systems with nonintrusive, data-driven reduced-order modelling approaches, thereby broadening the range of applications that can be addressed effectively.

Generative modeling of QG solutions
Louis THIRY (Sorbonne Université)
joint work with Petar Samardzic
In this talk, we will introduce denoising diffusion models, a class of generative models that rely on additive Gaussian white noise denoising. We will explain the link with particle-based methods of the heat equation in high dimension. We will apply these techniques to numerical simulations of the multi-layer QG equation in a double-gyre setting, which is an idealized model of the north-atlantic ocean with a western-boundary (gulf-stream like) current, viewing the numerical solutions of QG equations as a stochastic process that we learn without using explicitly physical priors.

Couette flow and rotation/stratification
Klaus WIDMAYER (University of Vienna & University of Zurich)
As is well known, perturbations of Couette flow in the 3d Navier-Stokes equations experience phase mixing, which stabilizes fluid motion. In the presence of suitable rotational forces or stratification, additionally dispersive internal gravity or inertial waves arise. These two mechanisms are of fundamentally different nature and relevant in complementary dynamical regimes. We will discuss how their combined effect leads to a quantitatively improved stability.
This is based on joint work with M. Coti Zelati and A. Del Zotto.

The deep-seated solar dynamo
Toby WOOD (Newcastle University, UK)
joint work with Devika Tharakkal, Craig Duguid, Paul Bushby
Since the advent of helioseismology, the strong rotational shear in the solar "tachocline" has been recognised as a likely site for toroidal field generation in the Sun. But the generation of poloidal field, which is essential to close the "dynamo loop", has generally been attributed to the overlying convection zone, and recent studies have even questioned whether the tachocline is involved in the dynamo at all. We present an alternative hypothesis, in which both the toroidal and poloidal field are generated in the tachocline by a combination of shear and magnetic buoyancy instability. We demonstrate a proof-of-concept Cartesian model and ongoing efforts to identify the effects of spherical geometry and other instabilities, such as MRI.

Scaling Trends in Rotating Convection
Paula WULFF (University of California, Los Angeles)
joint work with Jewel Abbate, Hao Cao, Jonathan Aurnou
Asymptotic analysis of rotating convective turbulence, as exists in planets and stars, yields two sets of scaling predictions for the limits of slowly rotating and rapidly rotating systems. These all hinge on the value of the convective Rossby number, Ro_c (e.g. Julien et al. 2012; Aurnou et al. 2020). Here, we test these asymptotic scalings using a broad compilation of laboratory-numerical rotating convection experiments in planar, cylindrical, and spherical systems. We find good agreement between the different rapidly $(Ro_c \ll 1)$ and slowly $(Ro_c \gg 1)$ rotating trends for convective heat transfer, velocities, length scales, and dynamic Rossby numbers in Boussinesq fluids. Further, we show that the predicted trends also hold in anelastic systems, albeit with an additional dependence on the density stratification. Our tests demonstrate that, given reasonable estimates of Ro_c in a geo- or astrophysical fluid system, accurate first-order predictions of the convective dynamics can be made.

Polar Vortex Crystals
William YOUNG (University of California, San Diego)
joint work with Lia Siegelman
Vortex crystals are quasiregular arrays of like-signed vortices in solid-body rotation embedded within a uniform background of weaker vorticity. Vortex crystals are observed at the poles of Jupiter and in laboratory experiments with magnetized electron plasmas in axisymmetric geometries. We computationally test the hypothesis that these organized structures, with vastly different space and time scales, can be reproduced by a maximally simplified 'quasigeostrophic' (QG) model of two-dimensional turbulence. The QG model shows that vortex crystals form from the free evolution of randomly excited two-dimensional turbulence on an idealized polar cap. Once formed, the crystals are long lived and survive until the end of the simulations (300 crystal-rotation periods). We identify a fundamental length scale characterizing the size of the crystal in terms of the mean-square velocity of the fluid and a parameter characterizing the variation of the Coriolis parameter close to the pole.

Stable regime singularity for the Muskat problem
Andrej ZLATOS (University of California, San Diego)
The Muskat problem on the half-plane models motion of an interface between two fluids of distinct densities in a porous medium that sits atop an impermeable layer, such as oil and water in an aquifer above bedrock. We develop a local well-posedness theory for this model in the stable regime (lighter fluid above the heavier one) that includes considerably more general fluid interface geometries than even prior whole plane results, and crucially allows the interface to touch the bottom. The latter can be used to model physically relevant scenarios where the heavier fluid invades a region occupied by the lighter fluid along the impermeable layer. We also show that finite time singularities do arise in this setting, including from arbitrarily small smooth initial data, by obtaining "maximum principles" for the height, slope, and potential energy of the fluid interface.

4 Posters

C brackets for shallow-water sloshing
Hamid ALEMI ARDAKANI (University of Exeter)
The second-order energy- and potential-enstrophy-conserving numerical scheme introduced by Arakawa & Lamb (1981) for the shallow-water equations with periodic boundary conditions, and extended by Salmon (2004) in the context of Hamiltonian Poisson-bracket discretisation, is further extended to a fourth-order discretisation for the problem of nonlinear shallow-water sloshing over a corrugated bottom surface in a rectangular rigid basin, with non-symmetric porous side walls and non-periodic inflow-outflow boundary conditions, with a biharmonic dissipation term. Adaptation to a finite domain with non-periodic inflow-outflow boundary conditions requires a new approach to the boundary conditions at porous solid boundaries, and the ghost cell grid point approximations in the context of the fourth-order finite-difference discretisation on the Arakawa C-grid. The scheme is implemented, shown to preserve (some) Casimirs of the Poisson bracket over long-time integration with bounded fluctuations.

On the size & shape of condensing steam
Bruno ANDREOTTI (LPENS)
Droplet-laden cloud dynamics emerge from phase transitions coupled with turbulent mixing. We establish scaling laws for steam-injected clouds in cool, dry air with condensation nuclei. Two asymptotic regimes arise: a jet-dominated regime, where cloud dimensions scale with injector radius, and a buoyancy-driven plume regime, where size follows a $\dot{m}^{2/5}$ scaling. Our mean-field analysis, based on the simplifying assumption that inter-droplet air remains humidity-saturated, successfully predicts cloud persistence governed by ambient enthalpy-humidity mixing. These findings bridge microphysical processes and macroscale dynamics, offering a quantitative framework for atmospheric cloud evolution.

Scalings of Moist Convection under Climate Change
Gregory DRITSCHEL (University of Leeds)
joint work with Steve Tobias, Douglas Parker, Lorenzo Tomassini
In order to explore the effects of climate change on atmospheric convection and the water cycle, we develop and analyse an extension of the Rainy-Bénard model, which is itself a moist version of the Rayleigh-Bénard model of dry convection. Including moisture changes the character of the convection, with condensation providing a source of buoyancy via latent heating. The Rainy-Benard model is set up for climate change, and analysed across the climate parameter space by examining diagnostics of the model's basic state, and its stability, with Convective Available Potential Energy (CAPE) calculations and a linear stability analysis. We use the linear stability results to identify new parameters relevant for this moist convective system, and to understand how the linear instability responds to them. The steady states of the model act as radiative convective equilibrium states, and are used to assess changes of the upper bounds of moist convection under climate change.

alpha-modified CL equations using GLM
Philomène DUFOUR (University of Hamburg)
Understanding wave–mean flow interaction is a central problem in geophysical fluid dynamics.

The Generalized Lagrangian Mean (GLM) theory of Andrews and McIntyre (1978) provides a systematic decomposition into mean and fluctuating components, but generally leads to a non-divergence-free mean velocity. More recently, Gilbert and Vanneste (2018, 2024) proposed a geometric reformulation of GLM based on flow maps, allowing the construction of divergence-free mean velocities.

An important open question concerns the incorporation of turbulence models within the GLM framework. In this poster, we present a first step toward deriving alpha-modified Craik–Leibovich equations, describing the interaction between surface waves and mean currents in the presence of turbulence represented through an alpha-parametrisation inspired by Lagrangian-averaged models.

Counter-rotating spherical Couette flow
Juan Cruz GONZALEZ SEMBLA (PMMH CNRS Sorbonne)
joint work with Laurette Tuckerman
We simulate the flow between counter-rotating spheres between free-slip boundaries. Our approach retains spherical topology but neglects weak curvature, and uses reduced radial resolution in order to cover a large spherical surface at low computational cost. In plane Couette flow, a similar reduced model has successfully reproduced turbulent–laminar banded patterns, including their wavelengths and angles. Such patterns are expected to occur between counter-rotating spheres, since they are observed both between counter-rotating cylinders which resemble the equatorial regions, and between differentially rotating disks which resemble the polar regions, when the horizontal dimensions greatly exceed the gap thickness. For Re near 40, we observe axisymmetric vortices in a latitudinal range encompassing neither the poles nor the equator. For Re near 60, we observe the high gradients which are hallmarks of turbulent-laminar bands.

Long-time behavior of ocean toy models
Julien GUILLOD (Sorbonne Université and ENS Paris)
joint work with Anne-Laure Dalibard, Julie Deshayes, Blandine Gorce, Antoine Leblond, Etienne Meunier, and Jacques Sainte-Marie
Simulating the evolution of the global ocean requires, in particular, determining initial data from which to begin the numerical simulation. The lack of available data, particularly at depth, makes this task very difficult. Currently, the initial state of the ocean is determined by integrating a numerical model over several thousand years until a kind of stabilization is observed; the actual numerical simulation is then launched from there. This spin-up procedure is unsatisfactory for both scientific (this stabilization depends on many choices) and ecological (majority of computing time used to determine the initial condition) reasons.

In order to better understand the important processes influencing the state of stabilization and to determine it more quickly, highly simplified models will be presented, for which the long-time behavior can be studied mathematically or numerically.

Energetics in sorted density coordinates
Benedict HATTON (LMD, École polytechnique)
joint work with Thomas Dubos, Rémi Tailleux
To study the energetics of stratified turbulent mixing, we use a sorted density coordinate to define the local available potential energy (APE). Averaging a tracer transport equation over isosurfaces of this sorted coordinate leads to a one-dimensional model of diapycnal tracer diffusion, with the diffusivity linked to the averaged dissipation of APE into BPE. In this work, we further investigate the connection between diapycnal diffusion and energetics by deriving 1D budgets of local turbulent kinetic energy (TKE) and turbulent available potential energy (TAPE), averaged over sorted density isosurfaces. Using an energetically consistent Boussinesq model gives access to constraints imposed by the first and second laws of thermodynamics, from which we can propose consistent turbulent closures. Extending the typical TKE-based approach to include TAPE may lead to a richer description of stratified mixing energetics.

Conservative discrete flows
Antoine LEBLOND (Max Planck Institute for Meteorology)
joint work with Peter Korn
We identified a discrete calculus approach for finite volume on collocated and staggered unstructured grids, and apply it to geophysical flow equations in order to obtain a hierarchy of discrete models covering fluid regimes relevant to general circulation models (GCM). This calculus provides a unified framework to explore and compare numerous discretisations, eases the analysis of invariants preservation, and leaves room for asymptotic preserving schemes. The Hamiltonian structure is transposed within this discrete setting. Some collocated-staggered correspondences enable the transfer of conservation properties on distinct grid types. These theoretical schemes are meant to be gradually implemented and tested within the GCM ICON developed at the Max Planck Institute for Meteorology.

Free Boundary 2D Navier-Stokes System
Paula LUNA VELASCO (Universidad de Sevilla – Universidad de Granada)
joint work with Francisco Gancedo, Eduardo García-Juárez

The Navier–Stokes equations describe the motion of incompressible fluids and continue to pose fundamental and challenging mathematical questions. In this talk, I will present new results on the two-dimensional free boundary problem for two immiscible fluids. We establish global-in-time well-posedness in the regime of nonnegative density and prove the persistence of the natural $C^{1+\gamma}$ regularity of the interface under sharp assumptions on the initial velocity. Moreover, we extend the analysis to the case of density-dependent viscosity and obtain global regularity when the viscosity contrast is sufficiently small.

Criticality and Navier-Stokes-Boussinesq

Maxence MANSAIS (Université Evry Paris Saclay)

In this talk I will discuss the construction of global in time mild solutions to the fractional Navier-Stokes-Boussinesq system (same fractional dissipation on the velocity and on the temperature) on the entire space in a critical setting for small initial data.

While a maximal critical space of resolution for the initial velocity is well known for the fractional Navier-Stokes system, obstacles exist to find the largest possible critical spaces of resolution simultaneously for the initial velocity, initial temperature and external force.

As an illustration of these obstacles, I will discuss and compare the framework of parabolic Morrey spaces applied to the fractional Navier-Stokes and to the fractional Navier-Stokes-Boussinesq system. I will also discuss some recent developments extending the previous parabolic Morrey spaces framework.

Exponential Mixing on the unit sphere

Marc NUALART (ICMAT-CSIC)
joint work with Augusto Del Zotto

In this talk we investigate the mixing of passive scalars on the two-dimensional unit sphere. We exhibit an incompressible velocity field that mixes exponentially fast any passively advected mean-free data. The velocity field consists of alternating two zonal flows with random amplitudes and can be thought of as the spherical analogue of the Pierrehumbert model on the torus.

On the Fritz John problem

Martin Oen PAULSEN (University of Bordeaux)
joint work with David Lannes

The Fritz John problem is a linear description of the interactions of an incompressible irrotational free surface fluid with a partially immersed solid object. The model was introduced by John in 1949 and is an important model closely linked to the Cummins equations, which are widely used by naval engineers to compute wave-structure interactions. However, despite their active use, it remains an open question whether the Cummins equations are well-posed. The main difficulty arises from the presence of corners in the fluid domain, which leads to the formation of singularities.

In this talk, I will present recent work in collaboration with David Lannes that addresses the well-posedness issue. Moreover, we study the singularities at the corners and their impact on the regularity of the solution.

On Stratosphere-Troposphere Coupling

Maria REBOREDO PRADO (University of Oxford)
joint work with Kathryn Gillow, Scott Osprey, Irene Moroz

Interactions between the stratosphere and troposphere have been identified as an important source of atmospheric predictability on timescales ranging from weeks to months. Yet, the dynamics and the relative contributions of the processes responsible for these interactions are not fully understood. To address this challenge, it is useful to develop minimal modelling frameworks to isolate and examine the mechanisms that couple the two layers.

We focus on the direct downward influence of stratospheric polar vortices on tropospheric flow. To do so, we consider a zonally symmetric quasi-geostrophic system in which the stratosphere and troposphere are represented as two domains with distinct static stability. The displacement of their interface, the tropopause, emerges naturally as part of the flow response to a forcing.

We present a set of experiments that show the response of this model to anomalous wave forcing at the stratosphere and compare it with the full observed response.

Modelling transitional boundary layer

Maharun Nesa SHAMPA (BTU Cottbus-Senftenberg)
joint work with Marten Klein, Heiko Schmidt

Understanding and modeling the Atmospheric Boundary Layer (ABL) remains challenging due to its strong variability and the continuous interaction between surface processes and large-scale atmospheric forcing. These difficulties become particularly pronounced in transitional flow regimes, where commonly used parameterizations often struggle to capture the underlying physics accurately. In this work, we focus on a simplified representation of the ABL, the neutral Ekman Boundary Layer (EBL), to better understand these challenges. We use a stochastic One-Dimensional Turbulence (ODT) model as a standalone tool to simulate this system and explore its capabilities across a range of Reynolds numbers. Our results show that the model is able to reproduce key features of the boundary layer, including velocity structure, wind turning, and surface-layer quantities such as friction velocity, with good agreement to reference data.

Effects of basin wind-stress on the MOC
Jonathan TESSIER (Institut des sciences de la mer, Université du Québec à Rimouski)
joint work with Louis-Philippe Nadeau, Malte Jansen
The ocean meridional overturning circulation (MOC) is a fundamental component of the Earth's climate system. Current conceptual theory describes the MOC as the interaction between a southern channel, a northern convection region, and a diffusive basin in between that balances excess mass transport with uniform upwelling through flat isopycnals. While observations and high-resolution models suggest that the MOC is linked to the shallow overturning cells associated with subtropical gyres, this coupling is not represented in the current conceptual framework of the MOC. In a recent study, we introduced a new zonally averaged model of the MOC that allows for meridional variations in basin density, defining the basin overturning through a local thermal wind balance at every latitude. Here, we extend the model to incorporate the effects of basin wind stress, to study the coupling between the MOC and the wind-driven gyres.

Equilibrium statistical physics of waves
Alexandre TLILI (CEA Paris-Saclay, Université Paris-Saclay)
joint work with Basile Gallet
We adapt the microcanonical framework of equilibrium statistical mechanics to predict the statistics of short linear waves in inhomogeneous moving media. For steady inhomogeneities and background flow, we compute the wave spectrum at any location in the domain based on an ergodic prescription for the action density in phase space, constrained by conservation of absolute frequency. We illustrate the method for near-inertial waves propagating through a geostrophic flow, for shallow-water waves subject to topographic inhomogeneities or to a background flow, and for deep-water surface capillary waves over a background flow, validating the predicted maps of wave kinetic energy, and root mean square surface elevation and interfacial slope against numerical simulations.

Convection in rotating spherical shell
Laurette TUCKERMAN (PMMH-CNRS)
joint work with Juan Cruz Gonzalez Sembla, Camille Rambert, Alan Riquier, Fred Feudel
For rapidly rotating geophysical flows, implicit numerical treatment of the Coriolis force allows much larger timesteps to be taken. By modifying a timestepping code to carry out Newton's method, we discover parameter regions in which the solution changes dramatically. We also carry out the first continuations in Ekman number with automatic adjustment of the spatial resolution.

Geostrophic internally heated convection
Yutong ZHANG (ETH Zurich)
joint work with Ali Arslan, Stefano Maffei, and Andrew Jackson
Convection on geophysical and astrophysical scales is subject to rapid rotation and internal heating. Analysis on long-time behaviors of such systems is challenging: energy identities do not capture effects of the Coriolis force, and extreme rotation rates make direct numerical simulations costly. Motivated by the non-hydrostatic quasi-geostrophic equations, we derive an asymptotically reduced model for rapidly rotating convection driven by uniform internal heating, in a plane periodic layer with isothermal, stress-free boundaries.

Within this model, we establish rigorous bounds on the long-time averaged thermal and kinetic energy dissipation rates, in terms of the Rayleigh and Ekman numbers, in the limit of infinite Prandtl number. Using the auxiliary function method, which formulates a variational problem constrained by energy balance relations, for both thermal and kinetic dissipation, we identify two distinct scaling behaviors above the onset of stationary convection.

5 Sponsors